PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

The Gravity Field of the Earth [and Discussion]

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The gravity field of the Earth

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In recent years the Earth's gravitational field has been determined with continually improving accuracy, by using hundreds of thousands of observations of Earth satellites, chiefly optical, laser and Doppler, together with surface gravimetry and, most recently, altimeter measurements from the Geos 3 satellite. The geopotential is usually expressed as a double series of tesseral harmonics, and several hundred of the harmonic coefficients are evaluated.

Progress in this work during the 1970s is briefly outlined, and some attempt is made to assess the accuracy of current geoid maps and sets of harmonic coefficients, as exemplified in the latest models derived at the Goddard Space Flight Center. The harmonic coefficients of order 14, 15 and 30 in the Goddard Earth Model 10B are compared with values obtained independently by analysis of resonant orbits: the results suggest that the values in GEM 10B are realistic for these orders, and presumably others. It appears that the accuracy of the geoid maps is now approaching 1 m.

1. Preamble

A comprehensive description of the Earth's gravity field cannot be fitted into a short paper. Here the aim is less ambitious: to give some idea of recent progress in the evaluation of the geopotential and to offer a few fragmentary answers to the question, 'How accurate are the values of the harmonic coefficients in recent geopotential models?'

In this paper, 'gravitational field' and 'geopotential' refer to the gravitational attraction due to the mass of the Earth and atmosphere. 'Gravity field' refers to gravity as measured at the Earth's surface, including the effect of the Earth's rotation.

2. Expression for the geopotential

It is usual to express the Earth's gravitational potential U at an exterior point (r, θ, λ) as an infinite series of tesseral harmonics in the form (Kaula 1966)

$$U = \frac{GM}{r} \sum_{l=2}^{\infty} \sum_{m=0}^{l} \left(\frac{R}{r}\right)^l P_l^m(\cos\theta)\{\bar{C}_{lm}\cos m\lambda + \bar{S}_{lm}\sin m\lambda\}N_{lm}, \tag{1}$$

where r is the distance from the Earth's centre, θ is co-latitude, λ is longitude (positive to the east), GM is the gravitational constant for the Earth (398600 km^3/s^2), R is the Earth's equatorial radius (6378.1 km), $P_l^m(\cos\theta)$ is the associated Legendre function of order m and degree l, and $\bar{C}_{lm}$ and $\bar{S}_{lm}$ are the normalized tesseral harmonic coefficients, which require to be evaluated. For $m \geqslant 1$ the normalizing factor N_{lm} is given by

$$N_{lm}^2 = 2(2l+1)\,(l-m)!/(l+m)!. \tag{2}$$

For $m = 0$, however, $N_{l0}^2 = 2l+1$.

Other representations of the geopotential are possible, but this format has proved to be most convenient in studies using satellites. In practice the series is arbitrarily truncated at the order and degree beyond which is it not thought possible to determine meaningful values of the harmonic coefficients $\bar{C}_{lm}$ and $\bar{S}_{lm}$. In recent years the truncation has been made at orders between 16 and 36, but larger arrays of coefficients may be used in future.

Equation (1) gives the exterior gravitational attraction due to the mass of the Earth and atmosphere. If the gravity felt at the Earth's surface is to be calculated, a 'centrifugal potential' $\frac{1}{2}r^2\omega^2\sin^2\theta$ has to be added to U, where ω is the Earth's angular velocity (72.92115×10^{-6} rad/s), and U should be the potential of the Earth excluding the atmosphere.

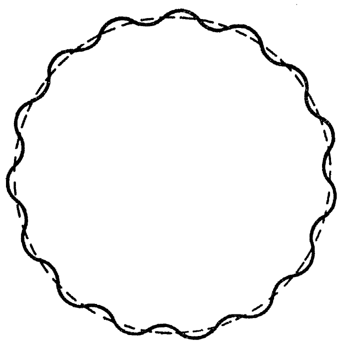

FIGURE 1. Section of Earth through the equator or any fixed latitude: form of longitudinal variation of geoid height due to 15th-order harmonic.

To obtain a pictorial expression of the harmonics in equation (1), it is useful to think of the suffix m as specifying variations from one *m*eridian to another. A harmonic of order m (for any degree $l \geqslant m$) exhibits m undulations as the longitude λ increases by 360°, for a fixed latitude, as shown in figure 1 for $m = 15$. For any fixed value of m, the suffix l determines the variation from one *l*atitude to another. A harmonic of degree l and order m has $(l-m)$ zeros in going from pole to pole along a fixed longitude, excluding zeros at the poles. If $m = 0$, there are no zeros at the poles, and a section of the Earth through the poles would exhibit l humps in 360° due to the $(l, 0)$ harmonic. If $m(>0)$ is even, there are zeros at the poles which are maxima or minima, and, including these in the count, a slice through the poles would show $(l-m+2)$ humps in 360°. If m is odd, the polar zeros are neither maxima nor minima (being points of inflexion), so the number of humps is reduced to $(l-m+1)$.

3. The Smithsonian Standard Earth II

Coefficients of some tesseral harmonics in the Earth's gravitational field were successfully evaluated in the 1960s, particularly those of low order, but the first satisfactory comprehensive model was the Smithsonian Standard Earth II published in 1970 by the Smithsonian Astrophysical Observatory, Cambridge, Massachusetts (Gaposchkin & Lambeck 1971). This model, which was a great improvement on its predecessors, relied largely on 100000 optical observations of satellites from Baker–Nunn cameras, with an observational accuracy of about 10 m. The expansion of the geopotential was truncated at degree and order 16, so that there were about 250 geopotential coefficients to evaluate. The orbital perturbations caused by this truncated geopotential were calculated, and the values of the geopotential coefficients and station coordinates were then adjusted so that the observations, of 21 satellites from about 30 stations, achieved the best possible fit to these perturbed orbits, and also satisfied geometrical constraints for simultaneous observations. In effect, about 200000 equations were being solved by least squares for more than 300 unknowns, namely the geopotential coefficients and the station coordinates.

The shape of the gravity field given by the Smithsonian Standard Earth II is shown in figure 2. This map gives the contours of the geoid, or mean sea-level surface, relative to a reference spheroid having a flattening of 1 part in 298.255, and the heights are in metres. Stippled areas indicate regions where the contour heights are negative. The accuracy is of order 5 m. This map has the advantage of being easy to interpret: it indicates, for example, that if you swam along the equator from south of India, where there is a depression about 110 m, to north of New Guinea, where there is a hump of about 80 m, you would at the end of your marathon swim be about 190 m further from the Earth's centre than at the start, without ever going uphill. The contours

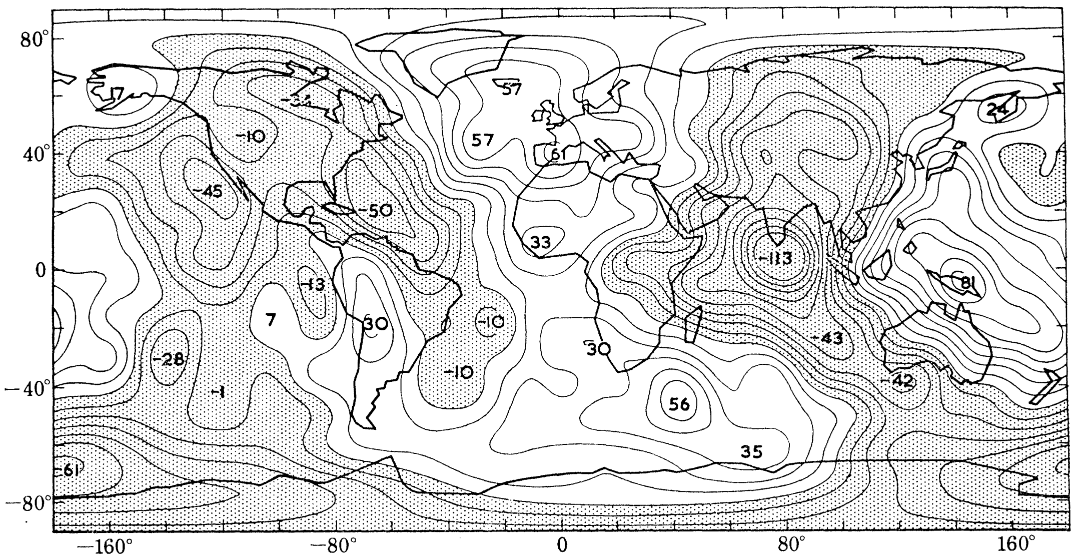

FIGURE 2. Geoid of Smithsonian Standard Earth II. Contours at 10 m intervals, relative to a spheroid of flattening 1:298.255.

define the rather peculiar shape taken up by the Earth's sea-level surface in response to the gravitational pull of the rather peculiar distribution of mass inside the Earth. Apart from the high area near New Guinea, there are also major humps near Britain and south of South Africa, both about 60 m high. The main depression is that south of India, but there are three others between 45 and 61 m deep, south of New Zealand and near Florida and California. Since the pull of gravity (if tides and transient effects are averaged) acts in a direction perpendicular to the mean sea-level surface, a geoid map like that of figure 2 gives probably the clearest pictorial representation of the gravitational field. In terms of gravity anomalies, that is the difference between the measured acceleration due to gravity and a reference field symmetrical about the equator, the picture looks rather different. The greatest negative anomaly, about −60 mGal†, is south of India, and the largest positive anomaly, about 40 mGal, is near New Guinea, but the other main features are more numerous and differently placed, with +33 mGal in Alaska and +32 mGal in eastern Europe, for example.

† 1 mGal = 10^{-5} m s^{-2}.

4. The GEM 10 geoid

Since 1970, many new gravitational field models have been published, including the Smithsonian Standard Earths III and IV (Gaposchkin 1974, 1977), the U.S. Department of Defense World Geodetic System 1972 (Seppelin 1974), the European models Grim 1 and 2 (Balmino *et al.* 1976*a*, *b*), and the series of Goddard Earth Models developed at the Goddard Space Flight Center, which have appeared in pairs, GEMs 1 and 2, GEMs 3 and 4, and so on, the latest being GEMs 10A and 10B (Lerch *et al.* 1978*b*). The observations used have gradually been extended during the 1970s. As well as the photographic observations, large numbers of Doppler measurements from the Navy Navigation Satellite System have been brought in, together with laser observations of ever improving accuracy, surface gravimetry of steadily increasing coverage, and, recently, radar altimetry from Geos 3, as well as smaller amounts of

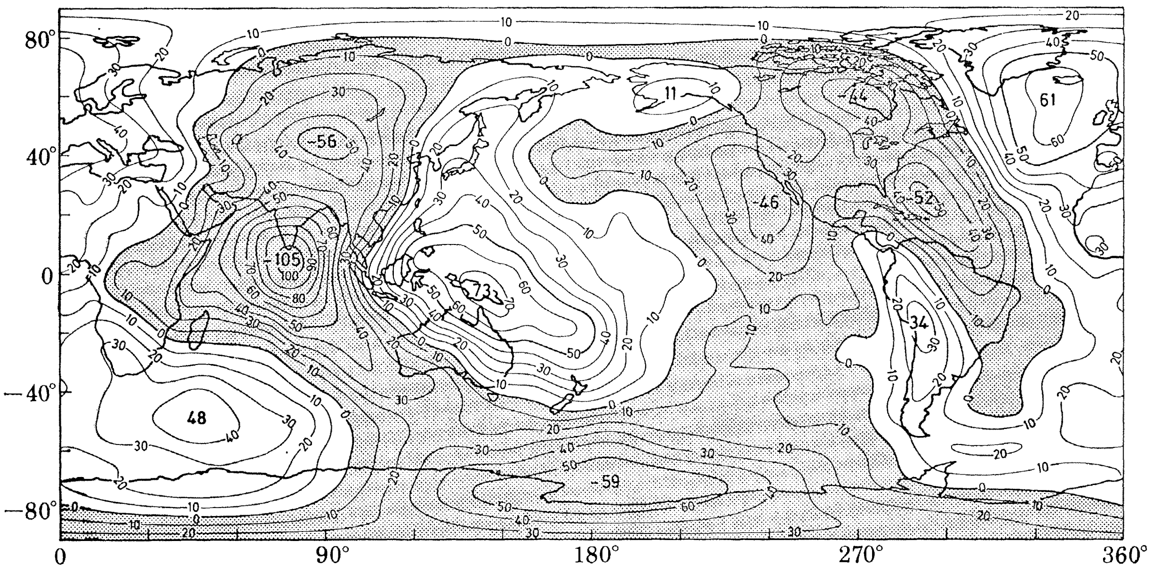

Figure 3. Geoid surface computed from the GEM 10 model. Height in metres above a mean spheroid of flattening 1:298.255.

other data. The geoid maps derived from these models fortunately all look fairly similar, and GEM 10, shown in figure 3, is probably the best available (in the absence of a GEM 10B map).

The GEM 10 model (Lerch *et al.* 1977) uses 840000 observations, including 213000 laser ranges, 150000 optical observations and 270000 U.S. Navy Doppler measurements. The most recent laser ranges are the most accurate of these. The solution also utilizes a set of 1654 equal-area 5° surface gravity measurements (Rapp 1977), which, although worldwide, are weak in the southern oceans. In all, 592 harmonic coefficients are evaluated in GEM 10, and the geopotential is complete to degree and order 22. Figure 3 shows the same main features as figure 2, although the world has been split at longitude 0° rather than 180°. The depression south of India is now 105 m deep as against 113 m in the earlier model, but the other three dips are quite similar, 59 m as against 61 m south of New Zealand, 46 m as against 45 m off California and 52 m as against 50 m off Florida. The New Guinea hump is 73 m as against 81 m. The hump near Britain is 61 m, the same, and that south of Africa is 48 m as against 56 m.

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GEM 10 is probably accurate to 1 or 2 m, if you accept that it inevitably smoothes out any fine detail, because even the 22nd harmonic has a semi-wavelength of 8°, or 900 km. The fine detail can best be appreciated by looking at the preliminary maps of the ocean surface obtained from altimeter measurements by the Geos 3 satellite (1975–27A): one of these is shown in figure 4 (Brace 1977). Here the detail is on a scale of a few kilometres, and the relative accuracy over small areas should be excellent, though the absolute accuracy over large areas must be treated with caution. The original maps are at contour intervals of 1 m, but for clarity only the contours at 5 m intervals are shown in figure 4. The contours in this section of the North Atlantic to the west of Europe differ in detail from those of GEM 10, being much more tortuous, but the general trends are similar. At a latitude of 60° N there is an increase of about 13 m between longitude

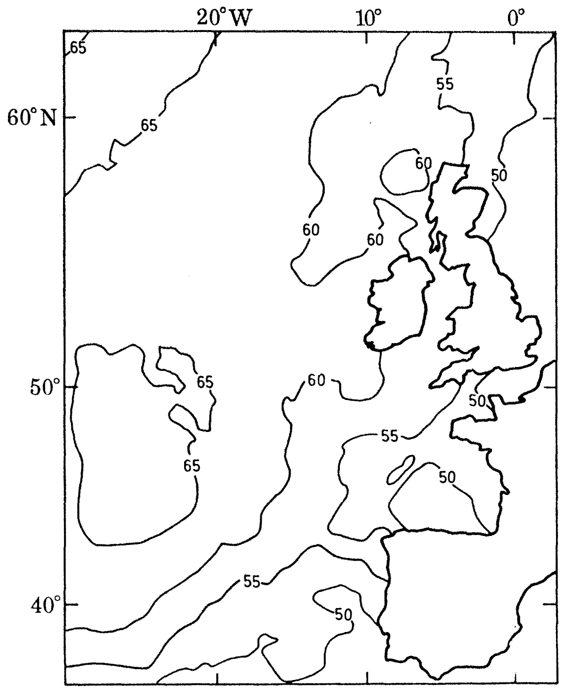

FIGURE 4. Preliminary geoid from Geos 3 altimetry; heights in metres. (From Brace 1977.)

0 and 30° W in the GEM 10 map (figure 3), and 15 m in figure 4. At longitude 30° W there is a decrease of only about 5 m in geoid height between latitudes 60° N and 40° N in GEM 10, and the value is the same in figure 4. The geoid in this area is very flat compared with many other areas of the world.

GEM 10 did not use altimetry data, so figure 4 is a nearly independent test and confirms the accuracy of the GEM 10 geoid. Current and future GEM models include altimetry data, which naturally tend to dominate the solutions for harmonics of order higher than about 30.

5. Accuracy of the harmonic coefficients

5.1. *The test of resonance*

Although the GEM geoid contours may be accurate to about 1 m (excluding fine detail), the accuracy of the 592 harmonic coefficients is still questionable. With more than a million equations

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to be solved for more than 600 unknowns, there are bound to be some very high correlations; or, to put it in another way, very different sets of coefficients might lead to nearly the same geoid. We need a correct set of coefficients not only to give the correct gravity field, but also because they would provide a strong indication of the mass distribution in the Earth's interior, supplying a criterion for judging between existing theories of the lithosphere and upper mantle of the Earth.

In the early 1970s the coefficients of order higher than about 10 were not very reliable, apart from a few for which there were orbits in shallow resonance; but recent models show great improvement. The accuracy of some high-order coefficients can be independently checked by analysis of orbits that pass through resonance with the Earth's gravitational field. 'Resonance' occurs when, after a certain number of revolutions, the satellite repeats its track over the Earth. For example, if the orbital period is such that the Earth spins through exactly 24° relative to the orbital plane between one equator crossing and the next, successive tracks of the satellite over the Earth will be 24° further west. After 15 orbits the ground track will have moved 360° and will then repeat itself. This is 15th-order resonance, and when it occurs the perturbing effects

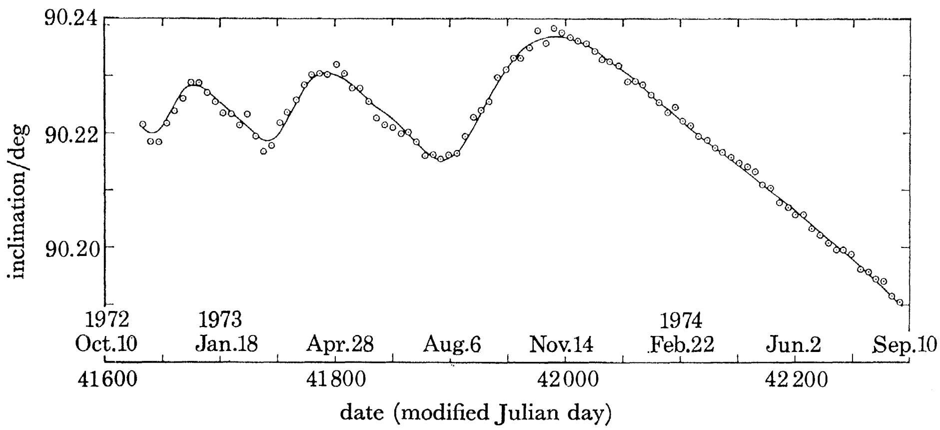

FIGURE 5. 1971–54A: values of inclination on approaching 15th-order resonance (⊙) with fitted theoretical curve (——).

of 15th-order harmonics in the geopotential build up day after day until there are quite large changes in some orbital parameters, particularly the inclination of the orbit to the equator. Accurate measurement of the change in inclination will give the value of a linear sum of harmonics of 15th order and odd degree, a 'lumped harmonic', as it is usually called. These resonances occur as the orbits slowly contract under the influence of air drag, and if the orbit is contracting slowly enough, the change in inclination can be very accurately fitted with the appropriate theoretical curve. Figure 5 shows the resonant variation of the satellite 1971–54A between November 1972 and September 1974 (King-Hele *et al.* 1975*b*). The inclination decreased by about 0.04°, equivalent to 5 km on the Earth's surface; measuring this major orbital change offers the opportunity of accurately determining lumped 15th-order harmonics.

By analysing a number of deep resonant orbits of this type, at different inclinations, values of individual 15th-order coefficients were obtained by King-Hele *et al.* (1975*a*, *b*); more recently, values of individual 14th-order coefficients of degree 14–22 have been derived (King-Hele *et al.* 1979). The values of the 14th and 15th-order coefficients in recent geopotential models will now be compared with these independent results.

5.2. *Coefficients of order* 14

Figure 6 shows the 14th-order *S*-coefficients of degree 14–22, as obtained by King-Hele *et al.* (1979): the values are shown as circles with error bars of length 2 standard deviations. The values are arranged so that they increase from left to right: the degree l of the coefficients, marked at the top, is consequently in a rather random order.

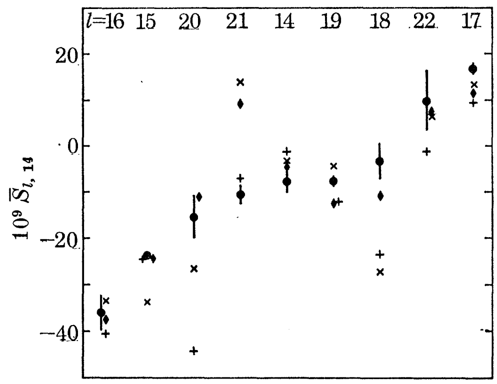

FIGURE 6. Values of $\bar{S}_{l,14}$. ●, King-Hele *et al.* (1979); ◆, GEM 10B; +, GRIM 2; ×, SSE IV.3.

Also shown in figure 6 are the values from three recent comprehensive geopotential models. The × signs are values from the Smithsonian Standard Earth IV.3 (Gaposchkin 1977); the + signs are values from the European model GRIM 2 (Balmino *et al.* 1976 *b*); the diamonds indicate values from the latest available GEM model. This model is not GEM 10, for which the geoid map was shown in figure 3, but GEM 10B (Lerch *et al.* 1978 *b*), which goes up to degree and order 36, incorporates the results of 700 passes of altimeter data from Geos 3, and is considerably better than GEM 10.

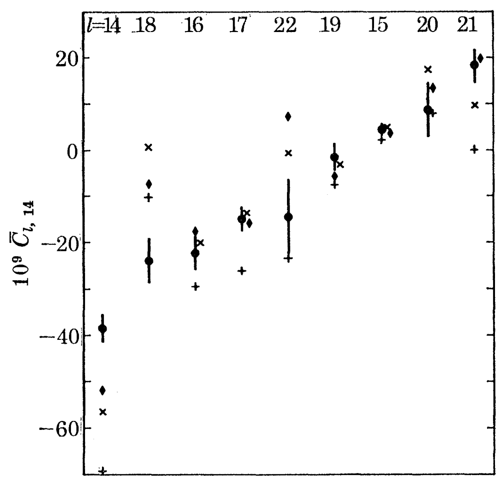

FIGURE 7. Values of $\bar{C}_{l,14}$. Symbols as figure 6.

For most values of l there is good agreement between the values of $\bar{S}_{l,14}$ in figure 6 from resonance, and those in the geopotential models. GEM 10B agrees particularly well for $l = 14$, 15 and 16, and differs seriously only for $l = 21$.

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Figure 7 shows the corresponding comparison for $\bar{C}_{l,14}$. Here the agreement with GEM 10B and SSE IV.3 is very good for $l = 15$ and 17, but the even-degree coefficients are less satisfactory. Of course, there is no certainty that the values from resonance are completely reliable; there is, however, no value of l for which all three models agree on a value of $\bar{C}_{l,14}$ different from the value obtained from resonance analysis, so it is very probable that the values from resonance *are* reliable, although their accuracy obviously needs to be improved by analysing further orbits.

It is not too surprising that the 14th-order coefficients in the comprehensive solutions are fairly accurate, because all the solutions incorporate results from satellites that are quite close to 14th-order resonance, and therefore suffer relatively large perturbations due to the 14th-order terms. A more severe test is the 15th-order coefficients, which we now examine.

5.3. *Coefficients of order* 15

In figure 8 the 15th-order *S*-coefficients derived by resonance analysis (King-Hele *et al.* 1975 *a*, *b*) are shown as circles with error bars, as before. For odd-degree coefficients the resonant solution was carried up to $l = 33$, and all the values are shown; but for the even-degree coefficients, only $l = 16$ and 18 are given in figure 8, because the two higher-degree coefficients

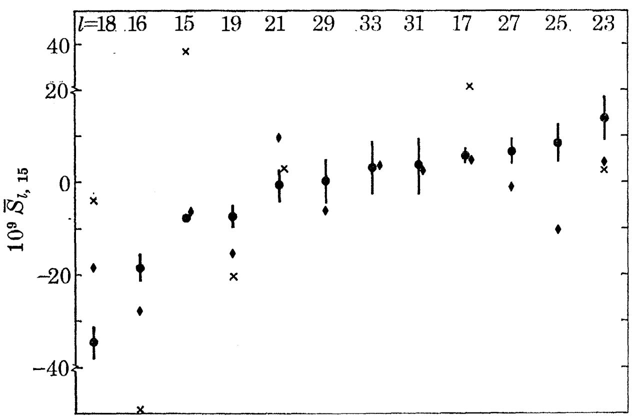

FIGURE 8. Values of $\bar{S}_{l,15}$. Symbols as figure 6.

evaluated ($l = 20$ and 22) were less accurate and less reliable. Again the values are arranged so that they increase from left to right. The values from GEM 10B and SSE IV.3 are shown for comparison. GRIM 2 is excluded because it uses the resonance results.

Figure 8 shows that, for 5 of the 12 values of l, the values of $\bar{S}_{l,15}$ from GEM 10B agree well with those from resonance analysis. The agreement is best for $l = 15$ and 17, and, perhaps surprisingly, for $l = 31$ and 33. Certainly the latest GEM model conforms to the resonance values much better than the GEM models that were current in 1975, when the resonance analysis was published. In GEM 10B the coefficients of order higher than 30 were determined solely from the altimeter data. Perhaps this is why the values of $\bar{S}_{l,15}$ for $l = 31$ and 33 agree so well with the values from resonance.

Figure 9 shows a similar diagram for $\bar{C}_{l,15}$. These results are better than those in figure 8. The agreement with GEM 10B is good for $l = 15$, 16, 17, 19, 23, 25, 29 and 33. It is probably fair to conclude that most of the coefficients in GEM 10B up to order 15, and possibly beyond, are accurate to within $\pm 5 \times 10^{-9}$.

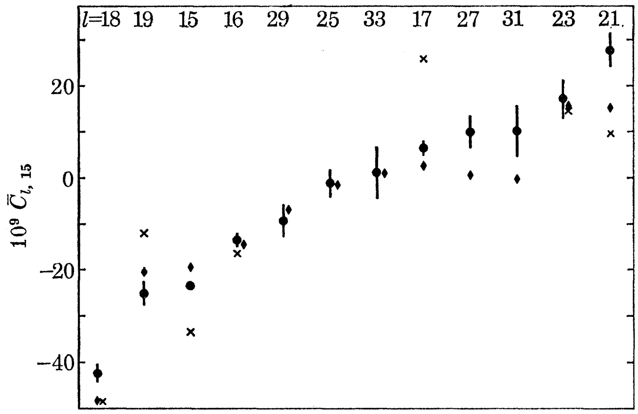

FIGURE 9. Values of $\bar{C}_{l,15}$. Symbols as figure 6.

5.4. *Coefficients of higher order*

The GEM 10B model includes harmonics to order and degree 36: is it possible to test the accuracy of any of its higher-order coefficients by resonance analysis? In principle, the answer is 'yes'. It should be possible to assess the accuracy of 29th and 31st-order harmonics by using 29:2 and 31:2 resonance. The first analysis of 29:2 resonance was made in 1976 by Doreen Walker, who succeeded in obtaining a lumped 29th-order harmonic coefficient from analysis of the orbit of Ariel 1 (1962o1) at 29:2 resonance (Walker 1977). Unfortunately, it is not possible to evaluate for comparison the corresponding lumped harmonic from GEM 10B, because coefficients up to degree 45 (or more) contribute to the lumped harmonic, and GEM 10B only goes to degree 36.

Analysis of the 31:2 resonance is more difficult, because of the high drag which is inevitable. The first lumped values were obtained by Hiller & King-Hele (1977) from the orbit of Proton 4 (1968-103A), and better values have recently been obtained from Skylab 1 rocket (King-Hele 1979). The equation obtained for the S-coefficients from Skylab 1 rocket (1973-27B) is:

$$0.011\bar{S}_{32,31}-0.083\bar{S}_{34,31}+0.310\bar{S}_{36,31}-0.701\bar{S}_{38,31}+\bar{S}_{40,31}-0.799\bar{S}_{42,31}+0.084\bar{S}_{44,31}$$
$$+0.508\bar{S}_{46,31}-0.372\bar{S}_{48,31}-0.192\bar{S}_{50,31}+\ldots=(-13.6\pm2.2)\times10^{-9}.$$

Although the lumped value is of good accuracy, it is not possible to obtain a value from GEM 10B for comparison, because values of the harmonic coefficients up to degree 50 or beyond would be required. The equation for the lumped C-coefficient has the same numerical factors on the left-hand side, and the value on the right-hand side is $(9.0\pm3.6)\times10^{-9}$.

Although the 29th and 31st-order coefficients in GEM 10B cannot yet be tested by this method, the 30th-order harmonics are open to verification. In the analysis of 15th-order resonance (King-Hele *et al.* 1975*b*) it was possible to obtain good values of lumped 30th-order harmonics from four satellites, and for one of these, the satellite 1971-54A shown in figure 5, the multiplying coefficients in the lumped harmonic decrease rapidly as the degree increases, so that only the coefficients of degree 30, 32, 34 and 36 are needed. The lumped S-coefficient from 1971-54A is given by

$$\bar{S}^{0,2}_{30}=\bar{S}_{30,30}+0.428\bar{S}_{32,30}+0.211\bar{S}_{34,30}+0.097\bar{S}_{36,30}+0(0.03\bar{S}_{38,30})=(15.3\pm1.3)\times10^{-9}.$$

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For the C-coefficient the corresponding value is

$$\bar{C}_{30}^{0,2} = (-10.3 \pm 1.5) \times 10^{-9}.$$

The values of the same lumped harmonic coefficients from GEM 10B are

$$\bar{S}_{30}^{0,2} = 11.2 \times 10^{-9} \quad \text{and} \quad \bar{C}_{30}^{0,2} = -8.4 \times 10^{-9}.$$

So, as shown in figure 10, the GEM 10B values are in the same direction as the values from resonance, and within 25% of the numerical value. Unless the agreement is just luck, it seems that the 30th-order coefficients in GEM 10B may be quite realistic.

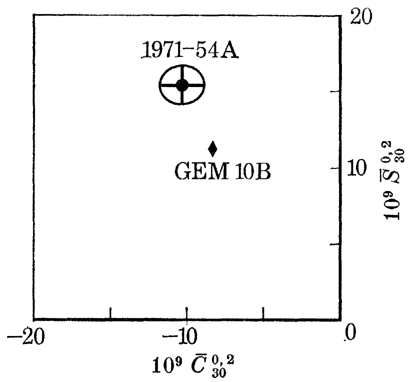

FIGURE 10. Lumped 30th-order harmonics from analysis of 15th-order resonance on 1971-54A, compared with values from GEM 10B.

5.5. *Conclusions*

The accuracy of the geopotential models has improved greatly in recent years. In the early 1970s the values of many of the harmonics of order greater than 10 were rather fictional, and the 15th-order coefficients determined from resonance differed widely from those in the models. But now there is good agreement with GEM 10B for order 14 and 15, and it is probably fair to conclude that the coefficients in GEM 10B should be accurate to $\pm 5 \times 10^{-9}$ (equivalent to about $\pm 20\%$ for order 15), for degree and order up to 15, and perhaps up to order and degree 30 or more, if the good agreement of the one result available for comparison is not due to luck.

6. VALUES OF *GM*

In addition to the values of the harmonic coefficients, the constant GM in equation (1) needs to be evaluated. The value is close to 398600 km³ s⁻², and until recently the best method of measuring this constant was from the trajectories of space vehicles. Values of $(GM - 398600)$ obtained from Mariners 9 and 10 and Vikings 1 and 2 respectively have been 0.66 ± 0.06 (Martin *et al.* 1975), 0.45 ± 0.2 (Esposito & Ng 1976), 0.40 ± 0.2 and 0.60 ± 0.2 (Esposito 1978). The unweighted average is 0.53. Values from lunar laser ranging are 0.48 ± 0.1 (Williams 1974) and 0.52 ± 0.03 (King *et al.* 1976). The most recent and probably the most accurate value, from laser tracking of near-Earth satellites, particularly Lageos (1976-39A), is $GM = 398600.44 \pm 0.02$ km³ s⁻² (Lerch *et al.* 1978*a*). All the values quoted above are from Lerch *et al.* (1978*a*), where the previous values have been adjusted to a consistent value for the speed of light, namely 299 792.458 km s⁻¹.

The value of the Earth's equatorial radius, R, from GEM 10B is 6378139 ± 1 m.

7. Future trends

During the 1970s the accuracy of the models of the Earth's gravity field has steadily improved, largely as a result of the improving accuracy of the satellite laser ranging measurements, now supplemented by altimeter data. In 1970 the geoid accuracy was 5–10 m. Today the figure is near 1 m, and the values of individual harmonics are quite realistic up to order 15 and possibly order 30. This continuing advance in geodesy contrasts with what may seem to be comparative stagnation in solid-Earth geophysics, where the simplistic concept of plate tectonics provided valuable insight in the late 1960s, but has tended to harden into dogma without maturing, so that, for example, Earth movements are often still modelled using rigid plates of constant thickness.

The advances in geodesy are likely to continue. There are many new methods of measurement that may help, for example (Marsh *et al.* 1977) satellite–satellite Doppler tracking and ranging. But progress is fairly certain without any new techniques, merely from the accumulation of more accurate data on laser ranging to satellites from the ground and a denser mesh of more accurate altimeter measurements. In the next Goddard Earth Model it is probable that harmonics up to order and degree 180 will be evaluated, so that there will be about 30000 geopotential coefficients to determine from millions of observations. In this 180×180 field the coefficients of order greater than about 30 will presumably be determined mainly from altimeter data. A geopotential field complete to order and degree 180 may be rather staggering, but a model with that degree of detail is obviously needed, because even the highest harmonics, of order 180, have a wavelength of 200 km, and the altimeter data show much finer detail. So the future promises more accurate and more detailed geoid maps, and a much fuller array of geopotential coefficients of improving accuracy.

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Discussion

N. A. G. Leppard (*Directorate of Military Survey, Feltham, Middlesex, U.K.*). How much does the determination of the higher-order harmonics (order greater than 100) depend on improving the accuracy of distance measurements to and from satellites?

D. G. King-Hele. The harmonics of order up to about 30 are likely to be determined mainly by ground observation of satellites and orbital perturbations. It seems probable that harmonics of order higher than about 40 will be determined primarily from measurements that give fine detail, such as altimeter data, and possibly various newer methods, such as satellite–satellite tracking between two satellites a few hundred kilometres apart in similar orbits. However, the full potentialities of the altimeter measurements cannot be utilized unless the orbit of the satellite is known as accurately as the height measurements. This can probably only be achieved if (i) numerous high-accuracy distance measurements are made (accurate to a few centimetres) within a few hours of the altimeter measurements, and (ii) the Earth's gravitational field is known with adequate accuracy, so that orbital perturbations can be accurately modelled.

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